"Mr. Lubbock was the first mathematician who undertook the extensive labors which such a conviction suggested [Empirical prediction of tide tables]. Finding that regular tide observations had been made at the London docks from 1795, he took nineteen years of these (purposely selecting the length of the cycle of the motions of the l nar orbit) and causes them to be analyzed by Mr. Dessiou, an expert calculator. He thus obtained tables for the effect of the moon's declination, parallax, and hour of transit, on the tides; and was enabled to produce tide tables founded upon the data thus obtained. . . in a very few years the tables thus produced by an open and scientific process were more exact than those which resulted from any of the secrets; and thus practice was brought into its proper subordination to theory." Page 90.

"The moon is distant from the earth's centre sixty times the earth's radius, and the attraction of gravity varies inversely as the square of the distance." Page 101.
We conclude then that the two overbalances at V and I, which will be called tide-generating forces, are nearly equal to one another, and vary inversely as the cube of the distance of the moon from the earth." Page 103. [Darwin explains the problem on page 101].

"But there is one case where inversion makes no difference; this is when the central spot is on the equator in the left hemisphere, for its inversion then makes the right hemisphere an exact reproduction of the left one. In this case
II therefore the two successive tides are exactly alike, and there is no diurnal inequality. Hence the diurnal inequality vanishes when the moon is on the equator." Page 156.

"The fact that the range of two successive tides is not the same is of great importance in tidal theory; it is called the diurnal inequality of the tide." Page 155.

"The higher tide occurs in that half of the daily circuit in which the moon passes nearest to the zenith or to the nadir of the observer." Page 156.

"It is also obvious that if the moon were twice as heavy as in reality, her tide-generating force would be doubled; and if she were half as heavy it would be halved. Hence we conclude that tide-generating force varies directly as the mass of the tide-generating body, and inversely as the cube of the distance." Page 157.

"The two bodies [sun and moon] are together at change of moon, and opposite at full moon. In both of these positions their actions conspire; hence at the change and the full of moon the tides are at their largest, and are called spring tides.
III When the two bodies [sun and moon] are at right angles to one another, it is half moon, either waxing or waning, the tides have their smallest range, and are called neap tides." Page 159.

"The observed facts agree pretty closely with this theory in several respects, for spring tide occurs about the full and change of moon, neap tide occurs at the
II half moon, and the range at springs is usually about three times as great as that at neaps. Moreover, the diurnal inequality conforms to the theory in vanishing when the moon is on the equator, and rising to a maximum when the moon is furthest north or south. The amount of the diurnal inequality does not, however, agree with the theory, and in many places the tide which should be the greater is actually less." Page 159

"It is a matter of rough observation that the tides follow the moon's course, so that high water always occurs about the same number of hours after the moon is due south. This rule has no pretension to accuracy, but it is better than no rule at all. Now at change and at full of the moon, the moon crosses the meridian at the same hour of the clock as the sun, for at change of moon they are together, and at full moon they are twelve hours apart. Hence the hour of the clock at which high water occurs at change and full of moon is in effect a statement of the number of hours which elapse after the moon's passage of the meridian up to high water. This clock time affords a rough rule for the time of high water at any other phase of of the moon; if, for example, it is high water at eight o'clock at full and change, approximately eight hours will always elapse after the moon's passage until high water occurs." Pages 160, 161.

"According to the equilibrium theory, high water falls at noon and midnight at full and change of moon, or in the language of the mariner the establishment of all ports should be zero. But observation shows that the establishment at actual ports has all sorts of values, and that in the Pacific Ocean (where the tidal forces have free scope) it is at least much nearer to six hours than to zero. High water cannot be more than six hours before or after noon or midnight on the day of full or change of moon, because if it occurs more than six hours after one noon, it is less than six hours before the following midnight; hence the establishment of any port cannot possibly be more than six hours before or after. Accordingly, the equilibrium theory is nearly as much wrong as possible, in respect to the time of high water. In fact, in many places it is nearly low water at the time that the equilibrium theory predicts high water." Page 161.

"The very difficult mathematical problem of the tides of an ocean covering the globe to a uniform depth was first successfully attacked by Laplace. He showed that whilst the tides of a shallow ocean are inverted at the equator, as proved by Newton, they are direct toward the pole. We have just arrived at the same conclusion by considering the tide wave in a canal in latitude 60°. But our reasoning indicated that somewhere in between higher latitudes and the equator, the tide would be of an undefined character, with an enormous range of rise and fall. The complete solution of the problem shows, however, that this indication of the canal theory is wrong, and that the tidal variation of level absolutely vanishes in some latitude intermediate
IV between the equator and the pole. The conclusion of the mathematician is that there is a certain circle of latitude, whose position depends upon the depth of the sea, where there is neither rise nor fall of tide." Page 177.

"It may seem strange that, whereas the first rough solution of the problem indicates an oscillation of infinite magnitude at a certain parallel of latitude, the more accurate treatment of the case should show that there is no oscillation of level at all. Yet to the mathematician such a result is not a cause of surprise. But whether strange or not, it should be clear that if at the equator it is low
IV water under the moon, and if near the pole it is high water under the moon, there must in some intermediate latitude be a place where the water is neither high nor low, that is to say, where there is neither rise nor fall." Page 178.

"Now let us take one more step toward actuality, and suppose the earth's equator to be oblique to the orbits of the moon and sun, so that they may sometimes stand to the north and sometimes to the south of the equator. We have seen that in this case the equilibrium theory indicates that the two successive tides on any one day have unequal ranges. The mathematical solution of the problem shows that this conclusion is correct. It appears also that if the ocean is deeper at the poles than at the equator, that tide is the greater which is asserted to be so by the equilibrium theory. If, however, the ocean is shallower at the poles than at the equator, it is found that the high water which the equilibrium theory would make the larger is actually the smaller and vice versa.

"If the ocean is of the same depth everywhere, we have a case intermediate between the two, where it is shallower at the poles, and where it is deeper at the poles. Now in one of these cases it appears that the higher high water occurs where in the other we find the lower high water to occur; and so, when the depth is uniform, the higher high water and the lower high water must attain to the same heights. We thus arrive at the remarkable conclusion that, in an ocean of uniform
V depth, the diurnal inequality of the tide is evanescent. There are, however, diurnal inequalities in the tidal currents, which are so adjusted as not to produce a rise or fall. This result was first arrived at by the great mathematician Laplace." Page 179.

"According to the equilibrium theory, when the moon stands some distance north of the equator, the inequality between the successive tides on the coasts of Europe should be very great, but the difference is actually so small as to escape ordinary observation. In the days of Laplace, the knowledge of the tides in other parts of the world was very imperfect, and it was naturally thought that the European tides were fairly representative of the whole world. When, then, it was discovered that there would be no diurnal inequality in an ocean of uniform depth covering the whole globe, it was thought that a fair explanation had been found for the absence of that inequality in Europe. But since the days of Laplace much has been learnt about the tides in the Pacific oceans, and we now know that a large diurnal inequality is almost universal, so that the tides of the North Atlantic are almost exceptional in their simplicity. In fact, the evanescence of the diurnal inequality is not much closer to the truth than the large inequality predicted by the equilibrium theory; and both theories must be abandoned as satisfactory explanations of the true conditions of affairs. But not withstanding their deficiencies both these theories are of importance in teaching us how the tides are to be predicted." Page 180.

"In the Pacific and Southern oceans the tidal forces have almost uninterrupted sway, but the promontories of Africa and South America must profoundly affect the progress of the tidal wave from east to west. The Atlantic Ocean forms a great bay in this vaster tract of water. If this inlet were closed by a barrier from the Cape of Good Hope to Cape Horn, it would form a lake large enough for the generation of much larger tides than those of the Mediterranean Sea, although probably much smaller than those which we actually observe on our coasts. . . The time of high water at any place must also depend on the varying depth of the ocean, for it is governed by the time occupied by the 'free wave' in traveling from the southern region to the north." Page 187.

"It is interesting to reflect that our tides to-day depend even more on what occurred yesterday or the day before in the Southern Pacific and Indian Oceans, than on the direct action of the moon to-day. But the relative importance of the two causes must remain a matter of conjecture, for the problem is one of insoluble complexity." Page 188.

"The moon, at her change, is close to the sun and crosses the meridian at noon; she would then be visible but for the sun's brightness, and if she did not turn her dark side toward us. She again crosses the meridian invisibly at midnight. At full moon she is on the meridian, visibly at midnight, and invisibly at noon. At waxing half moon she is visibly on the meridian at six at night, and at waning half moon at six in the morning. The hour of the clock at which the moon passes the meridian is therefore in effect a statement of her phase. Accordingly the relative position of the sun and moon is directly involved in the statement of the tide as corresponding to a definite hour of the moon's passage. A table founded on the time of the moon's passage must therefore involve the principal lunar and solar semidiurnal tides." Page 223.

"Indeed, in the language of sailors, the word 'tide' is not infrequently used as meaning tidal current, without reference to rise and fall. These currents are often of great violence, and vary from hour to hour as the water rises and falls, so that the pilot requires to know how the water stands in-shore in order to avail himself of his practical knowledge of how the currents will make in each place... It is, of course, still more important for ships to have a correct forecast of the tides where the entrance to the harbor is shallow." Page 6.

"The law of tidal currents in a uniform canal communicating with the sea is thus very different from that which holds in an open seacoast, where slack water occurs at high and at low water, instead of at mean water. But rivers gradually broaden and become deeper as they approach the coast, and therefore the tidal currents in the actual estuaries must be intermediate between the two cases of the open seacoast and the uniform canal." Page 57.

"In China the diurnal inequality is such that in summer the tide rises higher in the daytime than in the night, whilst the converse is true in winter. I suggest that this fact affords the justification for the statement that the summer tides are great." Page 77.

"Know that at different periods of the four seasons, and on the first and last days of the months, and at certain hours of the night and day, the seas have certain conditions as to the rising of their waters and the flow and agitation thereof." Zakariyya ibn Muhammad ibn Mahmud al Qazvini, "Wonders of Creation," pp. 103, 104. Wustenfield's edition. (Obiit 1283 A.D.)

"As to their monthly movement, he [Posidonius] says that the ebbs are the greatest at the conjunctions [of the sun and moon], and then grow less until the time of half moon, and increase again until the time of full moon, and grow less again until the moon has waned to half. Then the increase of the tide follows until the conjunction.

"The yearly movement of the tides he says he learned from the prople of Cadiz. They told him that the ebb abd flow alike were greatest at the summer solstice. He guesses for himself that the tides grow less from the soltice to the equinox, and then increase between the equinox and the winter solstice, and then grow less until the spring equinox, and greater until the summer solstice." Pages 82, 83.

"This [pages 82,83] is an execellent account of the tides at Cadiz, but I doubt whether there is any foundation for that part which was derived from hearsay. Lord Kelvin remarks, however, that it is interesting to note that inequalities extending over the year should have been recognized." Page 83.

"When the moon is in the equinoxes she is on the equator, and when she is in the solstices she is at her maximum distances to the north or south of the equator--or, as astronomers say, in her greatest north or south declination. Hence Seleucus means that, when the moon is on the equator, the tides follow one another with two equal high and low waters a day; but when she is distant from the equator, the regular sequence is interrupted. In other words, the diurnal inequality (which I shall explain in a later chapter) vanishes when the moon is on the equator, and is at a maximum when the declination is greatest. This is quite correct, and since the diurnal inequality is almost evanescent in the Atlantic, whilst it is very great in the Indian Ocean, especially about Aden, it is clear that Seleucus had watched the sea there, just as we should expect him to do from his place of origin." Page 85.

"The theory of tide-generating force which will be set forth in Chapter V is due to Newton, who expounded it in his 'Principia' in 1687. His theory affords the firm basis on which all subsequent work has been laid." Page 86.

"Nothing of importance was added to our knowledge until the great Frence mathematician Laplace took up the subject in 1774. It was he who for the first time recognized the difficulty of the problem, and showed that the earth's rotation is an essential feature in the conditions." Page 86.

Please note the change and check if wrong.

Return this table please.

Table 2. Arg (71)

To obtain value of argument at an earlier century apply one of the following 4 quantities so as to leave positive results.

$$\Delta = \begin{cases} -15.^{d}0 & -97.^{c}47 - a \\ -15.5 & +122.53 - a \\ +12.0 & +146.53 - a \\ +12.5 & -73.47 - a \end{cases}$$

Take a for the earlier date apply + numerical value of (a)

	a	Δ d	Δ c	corr	arg(71) d	arg(71) c		
−1700	−28.02				17.0	140.92		
−1800	28.74	−15.0	−68.73	0	2.0	72.19		
−1900	29.46	+12.5	−44.01	0	14.5	28.18		
−2000	30.17 [71]	+12.0	+176.70	0	26.5	204.88	×	Should be 204.88
−2100	30.88 [71]	−15.0	−66.59		11.5	138.29	×	
−2200	31.58 [70]	+12.0	+178.11		23.5	216.40	×	
−2300	32.28 [70]	−15.0	−65.19		8.5	251.21	×	
−2400	32.97 [69]	+12.5	−40.50		21.0	210.71	×	
−2500	−33.66 [69]	−15.0	−63.81		6.0	146.90	×	

Yr.	d		c			
−2500	6.0		146.90			
		−15.0		−63.81		
−2400	21.0		210.71		−115.69	−1.38
		+12.0		+179.50	+114.31	
−2300	9.0		131.21			−91.01
		−15.0		−65.19	−23.30	
−2200	24.0		96.40			+1.40
		+12.5		+41.89	−24.70	
−2100	11.5		138.29			+1.41
		−15.0		−66.59	−110.11	−22.58
−2000	26.5		204.88			
		+12.0		+176.70	+132.69	+107.97
−1900	14.5		28.18			
		+12.5		−44.01	−24.72	
−1800	2.0		72.19			
		−15.0		−68.73		
−1700	17.0		140.92			

Check again

Arg. 16

D in 36525 days = $\begin{cases} 1236 \text{ periods (lunations)} + 25.^{d}1932 \\ 1237 \text{ "} - 4.3374 \end{cases}$

Hence to obtain an earlier D argument one hundred years earlier.

$D_{-100} = D_0$ date $\begin{cases} + 4.^{d}3374 - a \\ - 25.1932 - a \end{cases}$ ("Century" period variation)

per. Cent. λ = 251^c
D = 18^c

a for earlier date.

but 18 × $\begin{cases} 1236 \text{ periods of D is } 22248.^{c} \text{ in Arg. 16} \\ 1237 \text{ " " D is } 22266. \text{ " " } \end{cases}$ 18^c

or more precisely

+ 25.^d1932

1236 periods of D = 88 (periods of 16) + 160.^c166 }
(per = 29.530588) 251^c +160
or 89 periods − 90.^c834 } 251^c

+91

1237 periods of D = 88 periods + 178.^c165 }
or 89 periods − 72.^c835 } 251^c

Hence when D difference is

$+ 4.^{d}3374 - a_D$ use for arg. 16 $\begin{cases} + 72.^{c}835 - a_{16} \\ - 178.165 - a \end{cases}$

$- 25.^{d}1932 - a_D$ use for arg 16 $\begin{cases} + 90.^{c}834 - a_{16} \\ - 160.^{c}166 - a \end{cases}$

	ΔD	a_{16}	(Δ16)	(16)
−1900		c		125.061
−2000	+4.3475	−.532	+73.367	198.428
−2100	+4.3477	−.545	−177.620	20.808
−2200	+4.3480	−.558	+73.393	94.201
−2300	+4.3482	−.571	+73.406	167.607
−2400	+4.3482	−.584	+73.419	241.026
−2500	+4.3487	−.596	−177.569	63.457
−2600	−25.1820	−.608	+91.442	154.899

Your values are sufficiently correct.

–Yr.	D d	(a)	–	1 c	(a)	2 c	(a)	3 c	(a)	4 c	(a)	5 c	(a)	6 c	(a)	7 c	(a)	12 c	(a)	16 c	(a)	17 c	(a)	18 c	(a)	19 c	(a)
2500 B	26.9236	113	3	10.996	+3	51.61	−37	95.73	+28	72.28	−58	53.74	−60	53.98	−31	99.20	+24	6.10	−3	63.	−597	4.83	+4	6.71	−6	51.00	+24
2400 B	22.5749	110		12.301	3	163.41	36	14.74	27	0.88	57	106.65		17.79	30	121.31	24	17.91	3	241.	584 = 13	44.36	4	25.06	6	56.15	24
2300 B	18.2265	108	2	13.606	3	119.22	35	49.74	27	53.51	56	30.59		113.63	30	52.44	23	5.72	3	167.	571 = 13	32.90	4	5.40	6	61.30	23
2200 B	13.8783	106	2	14.911	3	75.04	34	84.73	26	106.16	55	82.53		77.46	29	83.54	23	17.53	3	94.2	558 = 13	21.44	4	23.75	6	66.45	23
2100 B	9.5303	103	3	16.216	3	30.87	33	3.72	26	34.79	53	6.52		41.31	28	114.69	22	5.35	3	20.8	545 = 13	9.97	4	4.10	5	71.59	22
		2	2																	177.607		30.84		18.35		70.86	
2000 B	5.1826	−101		17.521	3	142.71		38.71	25	87.46	52	58.48		5.16	28	45.80	+22	17.16	−3	198.428		40.81	+4	22.45	−5	0.73	22
1900 B	0.8351	99	2	18.826	3	98.56		73.69	25	16.12		110.46		101.02	27	76.88	21	4.97	3	125.061	= 17.987	29.35	4	2.80	5	5.87	21
1800 B	26.0184	96	3	8.731	3	30.61		107.60		40.98		26.45 x		34.07	26	98.96	21	9.04	3	33.707	= 86.305	9.20	4	11.96	5	3.50	21
1700 B	21.6714	94	2	10.036	3	142.48		26.57		93.67		78.47		129.95	25	30.03	20	20.85	3	211.366	= 104.330	48.73	4	30.31	5	8.63	20
	4.3468	3	3																		= 0.014						
1600 B	17.3246	91		11.341	3	98.35		61.54		22.36		2.49		93.83	25	61.09	20	8.67	3	138.037		37.27	4	10.67	5	13.75	20
1500 B	12.9781	89	2	12.646	3	54.22		96.49		75.07		54.53		57.71	24	92.15	19	20.49		64.722	= 104.382	25.81	3	29.03	5	18.86	19
1400 B	8.6318	87	2	13.951	3	10.10		15.44		3.79		106.58		21.61	23	23.21	19	8.80		242.419	= 104.407	14.34	3	9.39	4	23.97	19
1300 B	4.2857	(3) 84	3	15.256	3	122.00		50.39		56.52		30.65		117.51	23	54.26	18	20.12		169.129	= 17.987	2.88	3	27.75	4	29.08	18
	= 25.1848 (2)		2																		= 18.014						
1200 B	29.4705	82		5.162	3	54.10		84.27		81.46		74.72		50.61	22	76.31	18	0.19		77.852		33.72	3	36.91	4	26.68	18
1100 B	25.1249	79	3	6.467	3	10.01		3.20		10.22		126.81		14.52	21	7.35	17	12.01		4.589	= 104.486	22.25	3	17.28	4	31.77	17
1000 B	20.7796	76	3	7.773	3	121.92		38.13		62.99		50.91		110.44	21	38.38		23.83		182.338	= 104.511	10.78	3	35.64	4	36.86	17
900 B	16.4346	74	2	9.078	3	77.85		73.05		115.78		103.03		74.37	20	69.41		11.66		109.100	= 0.014	50.31	3	16.01	4	41.95	16
		2	2																		= 104.565						
800 B	12.0898	72		10.383	3	33.79		107.96		44.58		27.17		38.31	19	0.43		23.48		35.876		38.84		34.37	4	47.03	16
700 B	7.7452	69	3	11.689	3	145.73		26.87		97.39		79.31		2.25	19	31.45		11.30		213.665	= 104.611	27.37		14.74		52.10	15
600 B	3.4009	(2) 67	2	12.994	3	101.68		61.77		26.21		3.47		98.20	18	62.47		23.13		140.467	= 18.006	15.90		33.11		57.17	14
500 B	28.5874	(3) 64	3	2.899	3	33.84		95.61		51.24		47.63		31.35	17	84.48 x		3.20		49.283	= 86.645	46.74		4.29		54.73	14
	4.3438	3	3																	177.829	= 104.672						
400 B	24.2436	61		4.205	3	145.80		14.50		104.09		99.82		127.31	16	15.48		15.03		227.112		35.26		22.66		59.78	
300 B	19.9001	59	2	5.510	3	101.78		49.38		32.96		24.02		91.28	16	46.48		2.85		153.955	= 0.013	23.79		3.03		64.83	
200 B	15.5568	57	2	6.815	3	57.77		84.26		85.84		76.23		55.26	15	77.47		14.68		80.811	= 0.013	12.31		21.41		69.88	
100 B	11.2137	54	3	8.120	3	13.76		3.13		14.73		0.46		19.24	14	8.46		2.57		7.680	= 104.752	0.83		1.78		74.92	
	4.3428		3																	177.883	104.780						
0 B	6.8709	51		9.425	Per = 141	125.76	Per = 156	38.00	Per = 116	67.63	Per = 124	52.70	Per. 128	115.23	(132°)	39.44	Per = 100	14.34	Per = 24	185.563	(18.00) Per = 251	40.36	Per = 51	20.16		3.95	Per = 76°

Per = 29.5306

How does −1.305 +10.094 = 11.400

.1901

(23.80)

51.61
7.42 + 44.19
119.22 − 111.80
75.04 + 44.18
30.87 + 44.17
111.80
142.71
44.18

Okay

(1.06)

(27.81)

100 = 104.96 = 52.26
= 83.74 x (8.01)
21.22 = 52.28
73.50 = 52.30
125.80 = 75.69
50.11 = 52.33
102.47 = 75.66
26.78 = 52.36
79.14 = 44.36
123.56 = 75.62
47.88

(30.81)

(9.00)

90.28
21.39 + 68.89
52.51 − 31.12
83.61 − 31.10
14.70 + 68.91
− 31.10
45.80
− 31.08

Okay

Follow sequence from x

2.17 = + 12.17
6.25 = − 4.08
18.08 = − 11.83
5.91 = + 12.17
(7.75)

112.460 73.103 / 91.089 = − 17.986
21.371

... try to insert 73.434 (3) for 2300?

0.77
0.97
0.95

28.88 = + 11.48 (8.69)
8.71 = + 20.17
= 48.22 = − 39.51

Is 2100 − 2500 correct?

Per. = 38

Put 2500 − Δ − 9.16 so that 15.90 follows 4.10 as in 500 and 900

(7.50)

29.5306

71			33			72			73		
d	c	(a)	d	c	(a)	d	c	(a)	d	c	(a)
5.5		-3368	26.5	80.36	-219	18.0	81.55	+1408	0.5	6.4	-189
21.0	209.15	102.53	22.0	12.20	215	23.0	72.37	1377	2.5	257.5	185
8.5	55.41	-24.72	18.0	42.01	210	28.5	62.90	1347	5.0	231.8	181
23.5	96.52	+23.30	13.5	71.87	206	2.0	95.73	1316	7.5	206.2	177
11.0	162.35	-134.17	9.0	3.85	201	7.0	85.66	1285	0.5	118.1	173
26.5	204.88	+132.69	5.0	33.75	-197	11.5	75.30	1254	3.0	93.5	-169
14.5	28.18	-24.72	0.5	65.65	192	16.5	64.64	1225	5.5	69.2	165
2.0	72.19	+23.28	26.0	3.60	188	21.5	53.69	1195	8.0	45.4	161
17.0	140.92	-104.37	21.5	33.59	183	26.5	42.44	1164	0.5	235.9	157
4.5	186.37	+102.91	17.0	63.63	179	31.5	30.88	1133	3.0	212.8	153
20.0	36.55	-24.72	12.5	93.71	175	4.5	60.01	1102	5.5	190.2	149
7.5	83.46	+23.26	8.5	25.83	170	9.5	47.84	1071	8.0	167.9	145
22.5	155.09	-98.03	4.0	56.00	166	14.5	35.35	1040	1.0	83.0	141
10.0	203.46	+97.05	29.0	92.21	161	19.5	22.55	1009	3.5	61.6	136
25.5	56.56	-0.74	25.0	24.47	156	24.5	9.44	978	6.0	40.5	132
13.0	106.41	-94.07	20.5	54.78	151	29.0	105.01	947	8.5	19.9	128
0.5	157.00	+92.56	16.0	85.14	146	2.5	23.28		1.0	213.7	124
16.0	12.34	-24.75	12.0	17.55		7.5	9.25		3.5	193.9	120
3.5	64.44	+23.24	7.5	48.01		12.0	103.90		6.0	174.5	115
18.5	141.29	-88.02	3.0	78.52		17.0	89.25		8.5	155.5	111
6.0	194.90	-86.50	28.5	17.08		22.0	74.30		1.5	74.0	107
21.5	53.27	-24.78	24.0	47.70		27.0	59.03		4.0	56.0	102
9.0	108.40	-83.42	19.5	78.37		0.0	84.46		6.5	38.3	98
24.0	188.31	+81.89	15.5	11.10		5.0	68.57		9.0	21.1	94
12.0	24.98		11.0	41.87		10.0	52.37		1.5	218.3	90
27.0	106.42	(220°)	6.5	72.70	(98°)	15.0 (31.5)	35.84	(109°)	4.0	202.0	(277°)
			2.5			(68°)	18.99			186.1	
			27.5				1.81			170.6	
							93.30			92.5	
							7.45			77.9	
							98.28			63.7	
							79.77			49.9	
							60.92			250.6	
							41.75			237.7	
							22.24			225.2	
							43.41			150.2	
							23.25			138.7	
							2.76			127.6	
							90.94			117.0	
							69.80			43.8	
							48.32			34.2	
							48.32				
							26.50				

74			76			77			
d	c	(a)	d	c	(a)	d	c	(a)	
10.0	47.0	-16	2.5	0.4	-52	4.5	26.6	+16	
5.5	49.0	16	0.5	13.1	51	3.0	39.7		16
1.0	50.3	16	6.0	42.5	50	1.5	63.9		16
12.0	35.6	15	4.5	55.8	49	0.0	12.1		15
7.5	37.0	15	3.0	9.9	48	8.5	25.2		15
3.0	38.4	15	2.0	23.2	47	7.0	38.3		15
14.0	23.8	14	0.5	36.6	46	5.5	51.4		14
9.5	25.3	14	6.5	6.1	45	4.0	64.3		14
5.0	26.8	14	5.0	19.6	44	3.0	12.3		14
0.5	28.3	13	3.5	33.3		1.5	25.2		13
11.5	13.9	13	2.0	47.1		0.0	38.1		13
7.0	15.5	13	1.0	2.0		8.5	62.0		13
2.5	17.1	12	6.5	31.1		7.5	9.8		12
13.5	2.8	12	5.0	45.2		6.0	22.5		
9.0	4.5	12	4.0	0.5		4.5	35.3		
4.5	6.3	11	2.5	14.9		3.0	48.0		
0.0	8.0	11	1.0	29.4		1.5	60.7		
10.5	64.8	11	7.0	0.0		0.5	8.3		
6.0	66.7		5.5	14.7		9.0	31.9		
1.5	68.6		4.0	29.6		7.5	44.4		
12.5	54.5		2.5	44.6		6.0	57.0		
8.0	56.4		1.5	0.7		5.0	4.4		
3.5	58.4		0.0	15.9		3.5	16.9		
14.5	44.4		5.5	46.2		2.0	29.3		
10.0	46.5		4.5	2.7		0.5	41.7		
5.5	48.6	(71°)	3.0	18.3 (7.0)	(59°)	9.5	0.0	(65°)	
							12.3		
							24.6		
							36.8		
							49.0		
							61.1		
							8.2		
							31.3		

(27.5 24°)

Check c in 2100 - 2400 what about 132.69 and 134.14

d   Table 2 Arg. (71)   27.5 (220)

-2500	6.0		146.90				
		-15.0		-63.81			
-2400	21.0		210.71		-115.69		
		+12.0		+179.50		-1.38	
-2300	9.0		31.21		+114.31		
		-15.0		-65.19		+91.01	
-2200	24.0		96.40		+23.30		
		+12.5		-41.89		-1.40	
-2100	11.5		138.29		-24.70		
		-15.0		-66.59✓		+1.41	
					-110.11		
						-22.58	-22.58
-2000	26.5		204.88				
		+12.0		+176.70			
-1900	14.5		-28.18		+132.69		
		+12.5		-44.01		+107.97	
-1800	2.0		72.19		-24.72		
		-15.0		-68.73		+1.44	
-1700	17.0		140.92		+23.28		
		+12.5		-45.48		-81.09	
					-104.37		
						+1.46	+2.90
-1600	4.5		186.37				
		-15.5		+149.82			
-1500	20.0		36.55		+102.91		
		+12.5		-46.91		78.19	
-1400	7.5		83.46		-24.72		
		-15.0		-71.63		+1.46	+2.92
-1300	22.5		-155.09		+23.26		
		+12.5		-48.37		-75.27	
					-98.53		-21.04
-1200	10.0		203.46			+1.48	
		-15.5		+146.90			
-1100	25.5		56.56		+97.05	+96.31	
		+12.5		-49.85			+2.98
-1000	13.0		106.41		-0.74	-93.33	
		+12.5		-50.59			
-900	0.5		157.00		-94.07		
		-15.5		+144.66		+1.51	-25.52
					+92.56		
-800	16.0		12.34			+67.81	
		+12.5		-52.10			
-700	3.5		64.44		-24.75		
		-15.0		-76.85		+1.51	+3.03
-600	18.5		-141.29		+23.24		
		+12.5		-53.61		-64.78	
-500	6.0		194.90		-88.02		
		-15.5		+141.63		+1.52	+3.06
					+86.50		
-400	21.5		53.27			+61.72	
		+12.5		-55.13			+3.08
-300	9.0		108.40		-24.78	-58.64	
		-15.0		-79.91			
-200	24.0		188.31		-83.42		
		+12.0		+163.33		+1.53	
-100	12.0		24.98		+81.89		
		-15.0		-81.44		+58.66	
					+23.23		-26.91
-0	27.0		106.42			-79.57	
		+12.5		-58.21	-102.80		
+100	14.5		-164.63				
				+161.01			
	2.5		3.62				

Table 2 Arg 16

Arg	D							(a)
			228.325	− 177.532	+ 104.090	+ 86.102		
	D		154.883	+ 73.442	− 17.988	− 68.151	+ 17.951	(a)
−2500	29.9236	−25.1820 − .0013	63.453	+ 91.430	− 86.139			−598
		+ 4.3487		− 177.569		− 18.025		
−2400	22.5749		241.022		+ 104.164			585
−2300	18.2265	+ 4.3484	167.603	+ 73.419	+ .014			572
−2200	13.8783	+ 4.3482	94.198	+ 73.405	+ .014			559
−2100	9.5303	+ 4.3480	20.807	+ 73.391	− 104.230	− .024		545
		+ 4.3477		177.621	+ 104.254			
−2000	5.1826		198.428	− (+ .024)		+ 86.267		532
−1900	0.8351		125.061	+ 73.367 (+ .076)	− 17.987	− 68.318	+ 17.949	519
−1800	26.0184	−25.1833 − .0015	33.707	+ 91.354	− 86.305	− 18.025		507
−1700	21.6714		211.366	− 177.659	+ 104.330			494
				+ 73.329				
−1600	17.3246		138.037		+ .014			481
−1500	12.9781		64.722	+ 73.315	− 104.382	− .025		468
−1400	8.6318		242.419	− 177.697	+ 104.407	+ 86.420		455
−1300	4.2857	−25.1848 − .0017	169.129	+ 73.290	− 17.987	− .027	− .052 (18.000 − .052 = 17.948)	442
				+ 91.277				
−1200	29.4705		77.852		+ 18.014	− 86.472		429
−1100	25.1249		4.589	+ 73.263	− 104.486	− .025		416
−1000	20.7796		182.338	− 177.749	+ 104.511			403
−900	16.4346		109.100	+ 73.238	+ .014			390
				+ 73.224				
−800	12.0898		35.876		− 104.565	− .026		377
−700	7.7452		213.665	− 177.789	+ 104.591	+ 86.605		363
−600	3.4009		140.467	+ 73.198	− 17.986	− 68.659	+ 17.947	350
−500	28.5874	−25.1865 − .0019	49.283	+ 91.184	− 86.645	− 18.027		337
				− 177.829	+ 104.672			
−400	24.2436		227.112					323
−300	19.9001		153.955	+ 73.157	+ .013			310
−200	15.5568		80.811	+ 73.144	+ .013			296
−100	11.2137		7.680	+ 73.131	− 104.752			283
				− 177.883		− .028		
−0	6.8709		185.563		+ 104.780	+ 86.794		269
+100	2.5284	−25.1884	112.460	+ 73.103	− 17.986	− 68.879	+ 17.945	255
200	27.7168		21.371	+ 91.089	− 86.835	− 18.028		242
300	23.3748		199.295	− 177.924	+ 104.863			228
				73.061				
400	19.0330		126.233	+	+ .013			214
500	14.6915		53.185	+ 73.048	− 104.918	− .028		200
600	10.3503		231.151	− 177.966	+ 104.946			186
700	6.0094		158.131	+ 73.020	+ .014			172
				+ 73.006				

The pencilled figures for 2200 – 2500 are empirical.
Thus 91.444 (2500) = 91.354 (1800) + .076 and
73.391 (2100) = 73.367 (1900) + .024.
Shall be interested to see how you produce "4 quantities" for Argument 16. Please be explicit. I am dumb!

Have checked all the figures several times.

Table 2 (Arg. 16) (18.000) (251)

			228.339	− 177.546	+ 104.090	+ 86.102			
	(13)	—	154.883	+ 73.456	− (17.988)	− 68.151	} + 17.951		
26.9234	− 25.1820	114	63.453	+ 91.430	86.139				−5.98
	+ 4.3487			− 177.569	−	− 18.025			
22.5747		111	241.022		+ 104.164			177.569	5.85
18.2265	+ 4.3484	109	167.603	+ 73.419	+ .014			5.85	5.72
13.8783	+ 4.3482	106	94.198	+ 73.405	+ .014			171.719	5.58
9.5303	+ 4.3480	104	20.807	+ 73.391	− 104.230			73.419	5.45
	4.3477			177.621		− .024		5.72	
	+			−	+ 104.254	+ 86.267		+ 250.858	
2000 5.1826	4.3475	101	198.428	(+.024) 73.367	− (17.987)		} + 17.949		−5.32
1900 0.8351	+ − 25.1833	99	125.061	+ (+.076) 91.354		− 68.318			519
26.0184	− (15)	96	33.707	+	− 86.305	− 18.025			507
21.6717		94	211.366	− 177.659	+ 104.330				494
				+ 73.329					
17.3246		91	138.037		+ .014				481
12.9781		89	64.722	+ 73.315	− 104.382	− .025			468
8.6318		87	242.419	− 177.697	+ 104.407	+ 86.420			455
4.2857	− 25.1848 (17)	84	169.129	+ 73.290	− (17.987)				442
				+ 91.277		− .027			
29.4705		82	77.852		+ 18.014				429
25.1249		79	4.589	+ 73.263	− 104.486	− .025			416
20.7796		76	182.338	− 177.749	+ 104.511				403
16.4346		74	109.100	+ 73.238	+ .014				390
				+ 73.224					
12.0898		72	35.876		− 104.565	− .026			377
7.7452		69	213.665	− 177.789	+ 104.591	+ 86.605			363
3.4009	− 25.1865 (19)	67	140.467	+ 73.198	− (17.986)	− 68.659	} + 17.947		350
28.5874		64	49.283	+ 91.184	− 86.645	− 18.027			337
				− 177.829	+ 104.672				
24.2436		61	227.112						323
19.9001		59	153.955	+ 73.157	+ .013				310
15.5568		57	80.811	+ 73.144	+ .013				296
11.2137		54	7.680	+ 73.131	− 104.752				283
				− 177.883		− .028			
6.8709		51	185.563		+ 104.780	+ 86.794			269
2.5284	− 25.1884	48	112.460	+ 73.103	− (17.986)	− 68.849	} + 17.945		255
27.7168		46	21.371	+ 91.089	− 86.835	− 18.028			242
23.3748		44	199.295	− 177.924	+ 104.863				228
				+ 73.061				(250.985)	
19.0330		41	126.233		+ .013				214
14.6915		38	53.185	+ 73.048	− 104.918	− .028			200
10.3503		35	231.151	− 177.966	+ 104.946				186
6.0094		33	158.131	+ 73.020	+ .014				172
				+ 73.006					
1.6687	− 25.1901	— 6	85.125	− 160.008	− 87.002	− .029			
26.8588			245.133		+ 87.031				
22.5187			172.156	+ 72.977	+ .013				
18.1788			99.192	+ 72.964	+ .015				
				+ 72.949					
13.8392			26.243		− 105.116	− .028			
9.4998			204.308	− 178.065	+ 105.144				
5.1607			131.387	+ 72.921	+ .014				
0.8219	− 19.5306	— 4	58.480	+ 72.907	− 54.907				
				+ 18.000					
20.3525			40.480		− 160.108				
16.0140			218.588	− 178.108	+ 105.231				
10.6757			145.711	+ 72.877	+ .014				
5.3377			72.848	+ 72.863	+ .015				
				+ 72.848	− 87.318				
0.0000	− 25.1932	—	0.000	160.166	—	− .029			
25.1932			160.166	− 72.819	+ 87.347				
19.8560			87.347	+ 72.804	+ .014				
14.5191			14.543	+					

Arg 33

−2500 26.5 79.93 219
+ 4.0 68.18
+)= 98.04 + 15
−2400 22.5 11.75 − 15 215
+ 4.5 − 29.86
−2300 18.0 41.61 210
+ 4.5 − 30.16
−2200 13.5 71.77 206
+ 4.0 + 68.03
−2100 9.5 3.74)= 98.04 + 15 201
+ 4.5 30.01 − 16
−2000 5.0 33.75 − 197
+ 4.5 − 31.90
−1900 0.5 65.65 192
− 25.5 + 62.05
−1800 26.0 3.60)= 92.04 188
+ 4.5 − 29.99
−1700 21.5 33.59 183
+ 4.5 − 30.04 (17)
−1600 17.0 63.63 179
+ 4.5 − 30.08
−1500 12.5 93.71 175
+ 4.0 + (67.88)= 98.05 + 14
−1400 8.5 25.83 170
+ 4.5 − 30.17 − 14
−1300 4.0 56.00 166
− 25.0 − 36.21 (15)
−1200 29.0 92.21 161
+ 4.0 + (67.74)= 98.05 + 15
−1100 25.0 24.47 − 15 156
+ 4.5 − 30.31
−1000 20.5 54.78 151
+ 4.5 − 30.36 (15)
− 900 16.0 85.14 146
+ 4.0 + (67.59)= 98.05 + 32 − 31
− 800 12.0 17.55
+ 4.5 − 30.46
− 700 7.5 48.01
+ 4.5 − 30.51 (16)
− 600 3.0 78.52
− 25.5 + 61.44)= 92.06
− 500 28.5 17.08
+ 4.5 − 30.62
− 400 24.0 47.70
+ 4.5 − 30.67 (16)
− 300 19.5 78.37
+ 4.0 + (67.27)= 98.04 + 29
− 200 15.5 11.10
+ 4.5 − 30.77 − 29
− 100 11.0 41.87
+ 4.5 − 30.83 (14)
0 6.5 72.70
+ 4.0 + 67.13
100 2.5 5.57
− 25.0 − 36.92)= 98.11
200 27.5 42.49
+ 4.5 − 30.97
300 23.0 73.46
+ 4.0 + 66.98)= 98.04 + 14 − 15
400 19.0 6.48
+ 4.5 − 31.06
500 14.5 37.54
+ 4.5 − 31.12
600 10.0 68.66
+ 4.0 + 66.84)= 98.05
700 6.0 1.82
+ 4.5 − 31.21 + 16 − 15
800 1.5 33.03
− 25.0 − 37.27
900 26.5 70.90
+ 4.0 + 66.68)= 98.05
1000 22.5 3.62
+ 4.5 − 31.37
1100 18.0 34.99